Online Appendix to Atoms for Peace, Redux

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This appendix outlines an alternative dynamic model for the baseline case that accompanies the paper "Atoms for Peace, Redux: Energy Codependency for Sustained Cooperation on the Korean Peninsula."

1 Alternative Setup for Baseline Model

In the baseline model introduced in the paper, we modeled a three-stage game between the North and the South, assuming nonstorable weapons production but endogenizing the transfer decision in the final stage. An alternative formulation of the final two stages will instead model the energy and weapons production choice as a dynamic optimization problem, assuming an exogenous transfer decision. We assume that weapons are now storable, such that the accumulation of weapons W is now given by

$$W_{t+1} = (1-\rho)\, W_t + Q_t, \tag{OA.1}$$

where Q is investment in weapons, and ρ is the depreciation rate for weapons, perhaps due to degrading or obsolescence. The (flow) resource constraint for the North is

$$(1-\alpha)\, P_{E_N,t} E_{N,t} + P_{W,t} Q_t = Y_{N,t}, \tag{OA.2}$$

where total Northern output Y_N is divided between production of energy E_N—a share $(1-\alpha)$ which is not transferred—and weapons, at the nominal prices of $P_{E_N,t}$ and $P_{W,t}$, respectively. Taken together, (OA.1) and (OA.2) imply

$$W_{t+1} = (1-\rho)\, W_t + \frac{Y_{N,t}}{P_{W,t}} - (1-\alpha)\, \frac{P_{E_N,t}}{P_{W,t}} E_{N,t}, \tag{OA.3}$$

Treating $E_{N,t}$ as the control and W_{t+1} as the state variable, the problem requires the solution to the dynamic program

$$\max_{E_{N,t}} \sum_{t=0}^{\infty} \delta^t U_N \left(E_S\right),$$

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subject to (OA.3), the initial condition $W_0 = 0$, and the no-Ponzi-game condition

$$\lim_{T \Rightarrow \infty} \left(\frac{1}{1-\rho} \right)^T W_{T+1} = 0,$$

where δ is the discount factor. The first order conditions are fairly standard:

$$\frac{P_{W,t}}{P_{E_N,t}} U'_N \left(E_{N,t} \right) = \delta \left(1 - \rho \right) \frac{P_{W,t+1}}{P_{E_N,t+1}} U'_N \left(E_{N,t+1} \right), \tag{OA.4a}$$

$$W_{t+1} = \left(1 - \rho \right) W_t + \frac{Y_{N,t}}{P_{W,t}} - \left(1 - \alpha \right) \frac{P_{E_N,t}}{P_{W,t}} E_{N,t}, \tag{OA.4b}$$

$$\lim_{T \Rightarrow \infty} \beta^T W_{T+1} = 0, \tag{OA.4c}$$

which are the (intertemporal) Euler, budget, and transversality conditions, respectively. If we let the utility function be given by the same linear additive functional form

$$U_{N,t} = \beta W_t + \left(1 - \beta \right) E_{N,t},$$

we can express (OA.4a) as

$$W_{t+1} = \frac{(1-\beta)}{\beta} \left[\frac{1}{\delta} \frac{P_{W,t}}{P_{E_N,t}} - \left(1 - \rho \right) \frac{P_{W,t+1}}{P_{E_N,t+1}} \right].$$

Generally, Q is indeterminate at any given time. If we impose the condition of interest, $W_{t+1} = 0$, the above simplifies to

$$\frac{1}{\delta} \frac{P_{W,t}}{P_{E_N,t}} = \left(1 - \rho \right) \frac{P_{W,t+1}}{P_{E_N,t+1}}.$$

To simplify the above expression, we now make the following assumption.

Assumption OA.1. (a) $P_{W,t} = P_W \ \forall t > 0$; (b) $P_{E_N,t} = P_{E_N} \ \forall t > 0$.

The two parts of the assumption impose constant prices for weapons and energy production in the North. Imposing Assumption OA.1 on the above equation, we obtain

$$\delta^* = \frac{1}{(1-\rho)}. \tag{OA.5}$$

This result is intuitive: There are no benefits to smoothing energy production through weapons accumulation if the present value of weapons, $\frac{1}{(1-\rho)}$, is equal to the discount factor, δ. Substituting (OA.5) into (OA.4b) then yields the optimal amount of energy production by the North in this case:

$$E_N^* = \frac{Y_N}{\left(1 - \alpha \right) P_{E_N}}. \tag{OA.6}$$

Note that (OA.6) is virtually identical to (A.2) in the proof for Proposition 1 in the text, with the exception that δ in (A.2) has been replaced by α in (OA.6). This is actually fairly sensible as well; since we have treated the transfer share α as exogenous, it shows up directly in the optimal energy production equation. For completeness, we re-state this variation of Proposition 1 below.

Proposition 1 (Atoms for peace redux (alternative)). *Given specific forms for North and South objective functions, and Assumption OA.1, if*

$$\delta = \frac{1}{(1-\rho)},$$

then the optimal investment by the South in the North is given by

$$\hat{I}^* = \frac{\chi}{P_I}\left(Y_S + \frac{\alpha}{1-\alpha}\frac{P_{E_S}}{P_{E_N}}Y_N\right) > 0. \tag{OA.7}$$

Proof. Follows directly from substituting (OA.6) for (A.2) in the proof of Proposition 1 of the text. □

2 Issues with Alternative Baseline Model

We find this setup more unsatisfactory than the baseline model presented in the paper, for several reasons.

1. This model leaves the transfer decision exogenous. We regard this to be a major shortcoming of this alternative approach, since an endogenous transfer decision is more general, and also lays out more clearly the specific conditions necessary to ensure that the equilibrium that we identified in the text exists.

2. As evident from (OA.7), the mathematics works out such that the key result of the model is little changed. While the alternative formulation of the model is indeed more elegant, the condition identified in Proposition 1 is a purely technical condition that has little relevance for policy.

3. By separating the production and transfer decision in the original model, we lay out the decisionmaking processes made by the North regarding weapons production and energy transfer—along with any additional associated assumptions—in a transparent fashion. The current model simply assumes a willingness by the North to make a transfer to the South, which in our view is by no means guaranteed after the energy has actually been produced.