Persistent Exchange Rate Volatility Despite Quasi-Stable Capital Flows

Jamus Jerome Lim and Cameron A. Shelton*

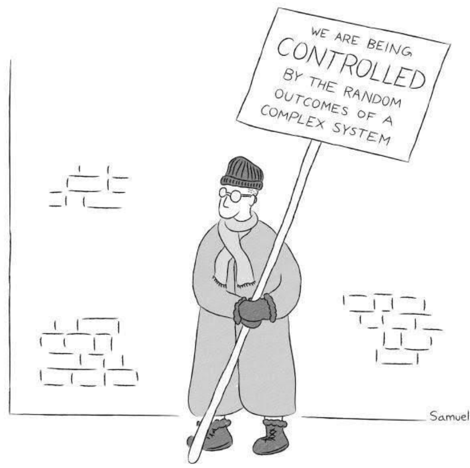
October 14, 2015

Abstract

This paper develops a two country, overlapping generations model with two assets, currency and capital. Boundedly rational agents allocate saving between a portfolio of the two assets, following strategies of imitation, experimentation, and election. Our main result is that, despite a convergence in the real interest rate and capital flows to a quasi-steady state, the exchange rate continues to exhibit persistent volatility. This result remains stable to a number of robustness checks, and indeed volatility is amplified if agents' propensity to experiment with new strategies is greater.

Keywords: exchange rate volatility persistence, agent-based models, genetic algorithm
JEL Classification: F41, G11, C63

*Abu Dhabi Investment Authority and Claremont McKenna College. Contact author: Jamus Lim, 211 Corniche St, PO Box 3600, Abu Dhabi, UAE, Email: jamus.lim@adia.ae. This note is a result of a joint project initiated at the Cognitive and Experimental Economics Laboratory, Trento, Italy. Financial support from the program in adaptive economic processes is gratefully acknowledged. We thank two anonymous referees, Jasmina Arifovic, and Peter Howitt for many insightful discussions but, of course, absolve them of any errors. The findings, interpretations, and conclusions expressed in this article are entirely those of the authors. They do not necessarily represent the views of the Abu Dhabi Investment Authority, its Directors, or the Government of Abu Dhabi. All simulations for the results in this paper were performed in Matlab, and are available at http://www.jamus.name/research/if11a.pdf. A copy of the code available at http://www.jamus.name/research/codeif11.txt.



1 Introduction

As has been noted by many authors—such as, *inter alia*, Mussa (1986) and Rogoff (1999)—the persistence of both nominal and real exchange rate volatility in the era of floating exchange rates has been a puzzle that cannot be easily explained within the conventional rational expectations framework. Attempts to do so have often resorted to assumptions of exogenous stochastic real shocks (Baxter & Stockman 1989) or of some form of nominal price rigidity (Chari, Kehoe & McGrattan 2002; Monacelli 2004).

However, another strategy for capturing such exchange rate volatility has been shown by a series of papers by Arifovic (1996, 2001). The approach entails modeling boundedly rational agents that apply a genetic algorithm to make portfolio allocation decisions. The key results of these papers have largely been in accord with empirical regularities: nominal exchange rate volatility, with persistence over time.

However, the elegant simplification of those papers—which limits the treatment of the economy to only the monetary side—obscures the potential role that real-side factors can play in influencing the path of the exchange rate. In particular, the presence of an equilibrium real interest rate for capital may serve to stabilize cross-border financial flows, and hence offer the possibility of greater stability in the exchange rate as well.

In this paper, we extend the basic two country, overlapping generations model that includes boundedly rational agents by incorporating a real side of the economy. In doing so, we embed elements that derive from the real side of the economy—as well as interactions between the portfolio allocation problem on the real and nominal side—that are absent from the essentially nominal Arifovic (1996) model.

Our results do not overturn the main Arifovic (2001) insight: that exchange rates remain persistently volatile, despite the presence of quasi-stable capital flows following a period of convergence. Moreover, this result cannot be simply attributed to the separation of investments within the portfolio via the genetic algorithm, since we allow for no-arbitrage conditions in the real exchange rate. Rather, the nominal exchange rate volatility is persistent because of the nature of the replicator dynamic for investment strategies, which is transmitted across generations as learned behavior.

We also find that while changing the relative likelihood with which a given portfolio decision is altered does not significantly alter our results, a reduced propensity of agents to experiment with their portfolio decision does give rise to significantly higher short run volatility; at the limit, this results in a period-by-period oscillation of the exchange rate between two values.

The central contributions of this paper are twofold: first, as in Arifovic (1996, 2001), it demonstrates the potential role that introducing an agent-based mechanism can play in accounting for observed international financial phenomena (in our case, the exchange rate). Second, it highlights the fact that stable equilibria in real markets may, in and of itself, be insufficient to generate stable nominal outcomes; in that limited sense, our work echoes the "sticky-price" literature (Chari *et al.* 2002; Monacelli 2004).

The recognition that asset returns tend to demonstrate volatility clustering and hence persistence in amplitudinal price changes goes back at least till Mandelbrot (1963). The by-now standard approach to capturing such variations in volatility has been to allow for conditional heteroskedasticity, á la ARCH-GARCH class econometric models (Bollerslev, Chou & Kroner 1992; Engle 2001). More recent efforts have sought to integrate such approaches with the market microstructure literature—see, for example, Andersen & Bollerslev (1997)—and, by and large, such models perform reasonably well when applied to exchange rates (Andersen, Bollerslev, Diebold & Labys 2001).

An alternative, econometrically-based approach to capturing the volatility persistence in exchange rates relies on Markov-switching processes (Engel 1994; Engel & Hamilton 1990). While such Markov-switching models are able to replicate the broader pattern of certain classes of exchange rates—in particular managed float regimes (Lee & Chen 2006)—the large swings generated as a result of the model's properties tend to lend themselves more to foreign exchange markets distinguished by a certain degree of intervention, rather than pure floats.

Agent-based models offer an alternative avenue for capturing volatility persistence in exchange rates without recourse to econometric methodologies. Consequently, such models offer theoretical mechanisms that can help explain the phenomenon, rather than (largely) theory-free econometric models. Agent-based models have found widespread purchase in modeling financial assets in general (Cont 2007), and exchange rates in particular (Arifovic 1996, 2001). Our work here rests squarely in this broader literature.

The rest of the paper is organized as follows. Section 2 presents a simple overlapping-generations model of the macroeconomy, and solves for the perfect-foresight equilibrium outcome as a benchmark. This is followed, in Section 3, by a discussion of the stochastic replicator dynamic and adaptation dynamics that are implemented to capture the behavior of boundedly rational agents in the portfolio allocation decision. We follow this with a presentation and discussion of the main results, including some basic robustness (Section 4), before a final section concludes with some brief policy implications.

2 An overlapping generations model with endogenous capital and wages

Consider a two-country, two-asset world with overlapping generations of agents and fiat money, similar to the environment introduced in Arifovic (1996, 2001), which in turn builds on the model of Kareken & Wallace (1981). For each time period t, N individuals are born in each country. Individuals live for two periods, supplying labor only when young. The labor supply is assumed to be inelastic, and for simplicity, is normalized to unity.

Consistent with Arifovic (1996), the consumption side is defined by loglinear preferences, and for an agent in generation t depends only on consumption, c, in both periods of life:

$$u_t\left[c_t\left(t\right), c_t\left(t+1\right)\right] = \ln c_t\left(t\right) + \ln c_t\left(t+1\right), \tag{1}$$

where the parentheses in (1) correspond to each period of life. A fully rational agent i in country j of generation t would solve the following optimization problem at time t:

$$\begin{aligned} &\max u\left[c_{ij,t}\left(t\right)\right] \quad \text{s.t.} \\ &c_{ij,t}\left(t\right) \leq w_j\left(t\right) + s_{ij,t}\left(t\right), \\ &c_{ij,t}\left(t+1\right) \leq s_{ij,t}\left(t\right)|_{postreturn}, \qquad i \in \mathfrak{I}, j = 1, 2, \end{aligned}$$

where w_j is real wages in country j (which agents take as given), and $s_{ij,t}$ is saving by agent i of generation t in country j. $s_{ij,t}|_{postreturn}$, the value of the agent's savings in the following period, is defined as

$$s_{ij,t}\left(t+1\right)|_{postreturn} \equiv \sum_{j=1}^{2}\left[\frac{m_{ij}\left(t\right)}{p_j\left(t+1\right)}\right] + \sum_{j=1}^{2}\left[r_j\left(t\right)k_{ij}\left(t+1\right)\right],$$

where m_{ij} are the money market holdings of agent i in currency j, p_j is the nominal price of goods in country j, r_j is real returns to capital in country j (which each agent also takes as given), and k_{ij} is the investment of the same agent i in the capital of country j. The second term in the above expression constitutes nominal returns to capital, and is an extension of our model relative to Arifovic (1996) and Kareken & Wallace (1981) (where savings are solely comprised of real currency holdings). In the optimum, for consumption when young given by wage income less saving, consumption when old will fully exhaust the return on savings (which includes both real currency and capital). Optimal saving is obtained by combining the Euler equation from the maximization problem above with the budget constraints evaluated at equality; this yields

$$s_{ij,t}(t) = w_j(t) - \frac{s_{ij,t}|_{postreturn}}{R(t)}, \tag{2}$$

where $R(t)$ is the discount factor.

We endogenize both real wages and capital (relative to our predecessor models) in a fairly straightforward way. Capital and labor are utilized by a large number of P identical, profit-maximizing firms, each of which face the (Cobb-Douglas-type) production technology

$$f[k(t)] = Ak(t)^{\alpha}, \tag{3}$$

where k is the capital in intensive form (the capital-labor ratio), A is a technology shifter, and $0 \leq \alpha \leq 1$ is the output elasticity of intensive capital (since firms are identical, we have omitted subscripts to denote each firm to simplify the exposition). Production is increasing in capital but exhibits diminishing returns, so that $f' > 0$ and $f'' < 0$. Capital is subject to no depreciation (the depreciation rate $\delta = 0$), so that for capital investment q,

$$k(t+1) = q(t) + (1-\delta)k(t) = q(t) + k(t).$$

Since firms are identical, the following (representative) profit maximization problem characterizes the firm problem at time t:

$$\max f[K_j(t)] - w_j(t) - r_j(t)Q_j(t+1), \qquad j = 1, 2,$$

where $K_j = \sum_P k_j$ and $Q_j = \sum_P q_j$ are the total capital stock and aggregate investment in country j, respectively (in this and what follows, aggregates of variables are denoted by their uppercase). Since firms are optimizing agents, the first order necessary conditions are the familiar

$$\begin{aligned} f'[K_j(t)] &= r_j(t), \\ f[K_j(t)] - K_j f'[K_j(t)] &= w_j(t), \qquad j = 1, 2, \end{aligned}$$

Imposing the functional form in (3) into the above allows us to rewrite the above as

$$\begin{aligned} \alpha f'[K_j(t)] &= r_j(t), \\ AK_j(t)^{\alpha} - \alpha AK_j(t)^{\alpha} &= w_j(t), \end{aligned} \tag{4}$$

In equilibrium, the exchange rate $e(t) = \frac{p_1(t)}{p_2(t)}$ is constant over time, since the following no-arbitrage condition must hold:

$$\frac{p_1(t)}{p_1(t+1)} = \frac{p_2(t)}{p_2(t+1)} = r_1(t) = r_2(t), \tag{5}$$

implying that $\frac{p_1(t)}{p_2(t)} = \frac{p_1(t+1)}{p_2(t+1)}$ for all t.

Assuming that there are no restrictions on trade and investment in each country, and that exchange rates are flexible, asset market equilibrium implies global aggregate savings $S = \sum_{J=2} \sum_{N} s_{ij}$ will equal the sum of real world money supply plus aggregate capital stock:

$$S(t) = \frac{N}{2}(w_1(t) + w_2(t)) = \sum_{j=1}^{2} K_j(t+1) + \frac{M_1(t)}{p_1(t+1)} + \frac{M_2(t)e(t)}{p_2(t+1)}, \quad (6)$$

where $M_j = \sum_N m_j$ is the national money supply of country j, and we have used the optimal saving result (2) evaluated with the discount factor set consistent with asset market equilibrium (5). With a constant money supply, $M_j(t) = M_j(0) = \bar{M}_j$ for $j = 1, 2$. This recovers celebrated result of Kareken & Wallace (1981): that the exchange rate is indeterminate, since perfect currency substitution implies that we have only a single currency market equilibrium condition from which to define the price levels in both countries. Because of this extra degree of freedom, if there exists any monetary equilibrium in which both currencies are valued at some exchange rate, e, we can always find an alternate sequence of prices to support a different exchange rate $e' \neq e$ under the same investment decisions.[1]

Consequently, for a given exchange rate $e \in (0, \infty)$, a stationary, perfect-foresight equilibrium exists with constant price levels, rates of return on each of the two currencies and capital stocks, and constant (Pareto-optimal) consumption allocations such that $c^*_{j,t}(t) = c^*_{j,t}(t+1)\ \forall j = 1, 2$.

3 Stochastic replicator dynamic and adaptation of strategies

While the exchange rate in the perfect-foresight benchmark introduced in Section 2 is indeterminate, the no-arbitrage condition (5) means that the exchange rate is constant over time. To allow for possible fluctuations in the exchange rate, we follow Arifovic (2001) and Sargent (1993) introduce bounded rationality into the benchmark model. In particular, we inject genetic algorithm dynamics into the model by relaxing (1).

More specifically, agents are now boundedly rational, and follow a simple portfolio allocation strategy, with decisions made as follows: a fraction a of an agent's wages are saved (conversely, a fraction $1a$ is consumed). Of the fraction saved, b is invested in currency and the remaining $1b$ in capital (this additional allocation between currency and capital is what distinguishes our portfolio allocation process from Arifovic (2001)). Further choices are made with regard to investment in currency and capital in country 1 (country 2), and these are given by c and d ($1-c$ and $1-d$), respectively. A schema summarizing the portfolio allocation decisions facing agents is shown in Figure 1.

[1]Since the result is secondary to our interests, we do not attempt a rigorous proof here. For an exposition of the proof without capital, see Arifovic (2001). The proof with capital is analogous, but with an additional capital term carried around.

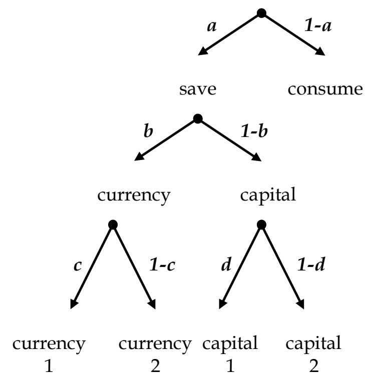


Figure 1: Portfolio allocation decisions for boundedly rational agents. The top nest is the saving-consumption tradeoff. Saving is then allocated between currency and capital, and subsequently between those of one country versus the other.

The fitness of each strategy equals the utility realized by the agent who played that strategy. The population of strategies evolves by the application of three operators: imitation, experimentation, and election.[2]

Imitation entails agents of generation $t+1$ choosing their strategy from the population of strategies employed by generation $t-1$. This imitation is governed by a proportional fitness rule, meaning the probability of choosing a particular strategy is proportional to its performance in the population two periods ago. Imitation thus allows existing rules with high fitness values to be copied more frequently.

After the new agents draw strategies from the previous population via the imitation rule, there is some chance that they will *experiment* on (at most) one of the portfolio decisions. Each new agent chooses one of the four portfolio decisions (captured in Figure 1) and, with probability π_x replaces the original fraction with a randomly generated fraction drawn from a $U[0,1]$ distribution. The other three portfolio strategies are left unchanged.

Experimentation thus constitutes an alternative strategy which is then tested against the original by means of an *election*. The election operator compares the fitness of the original and the alternative at the previous wages, prices, and rental rates. If the alternative strategy is of equal or greater fitness, it is accepted in place of the original. This rule effectively introduces random drift into the population of strategies.

The sequence of events are as follows: since capital is predetermined by the investments made by the young of last period, the labor income $w(t)$ of

[2]These follow genetic updating algorithms chosen by Arifovic (2001). Note that we have limited the application of the genetic algorithm to portfolio choices alone, and not consumption or production decisions in the economy.

this period's young is therefore independent of their investment decisions. This period's capital also determines the rental rates $r(t)$. This period's young will then use their income to make investment strategies—the fractions of which are represented by a, b, c, and d—into capital and currency instruments.

Their investments then determine this period's prices, as well as next period's capital. Prices and rental rates determine the returns on the saving of this period's old, and therefore their consumption. From this, the fitness of the strategies employed by this period's old is calculated. Finally, the strategies of next period's young spring from those of this period's old through the stochastic replicator dynamic.

4 Main results and discussion

Our baseline simulation is run for a population of $N = 30$ and timeline of $T = 500$. We set $M_1 = M_2 = w_1 = w_2 = 100$, allow equal probability of asset allocation ($a = b = c = d = 0.25$), and limit the probability of experimentation to $\pi = 0.3$.[3,4] The results are presented in Figure 2.

It is clear from the top left panel that the exchange rate continues to display properties of volatility persistence, consistent with the results in Arifovic (2001) and real-world data. Importantly, this is in spite of the fact that there is convergence of the interest rate to a more-or-less stable level around unity, which we term a "quasi-steady state," as well as a relatively stable (albeit slightly more volatile) capital stock in each country (which also implies stability in net cross-border capital flows); we likewise regard such capital flows as quasi-stable. Wages likewise converge to a quasi-steady state (around 300), moving in the opposite direction from interest rates (as expected since labor and capital are substitutes).

Our first set of experiments considers changing the parameters associated with the experimentation rule. We consider two alternatives: first, we increase the propensity of agents to experiment with the saving-consumption decision to half ($a = 0.5$), while equally distributing remaining probabilities ($b = c = d = 0.167$) (all other parameters follow the baseline). Second, we decrease the overall experimentation rate by an order of magnitude ($\pi = 0.03$). All other parameters retain their baseline assumptions. The key charts, corresponding to the exchange rate and capital, are shown in Figure 3.

Taken together, these perturbations yield an interesting result. Although altering the relative likelihood with which portfolio decisions are made results in little difference relative to the baseline,[5] the overall propensity to experiment

[3] All simulations were performed in MATLAB. The code used to generate the results is available online, at http://www.jamus.name/research/codeif11.txt.

[4] Note that the firm count P does not matter for the simulations, and is thus not reported. Since firms are identical, they will be price takers as long as P is large enough. Moreover, firms enter into the portfolio allocation process by providing investible capital, and so their specific count does not matter. For these reasons, the specific number of firms is irrelevant.

[5] This is the case even when we consider alternative permutations of portfolio allocation shares; for example, increasing either c or d to 0.5, and distributing the remaining shares

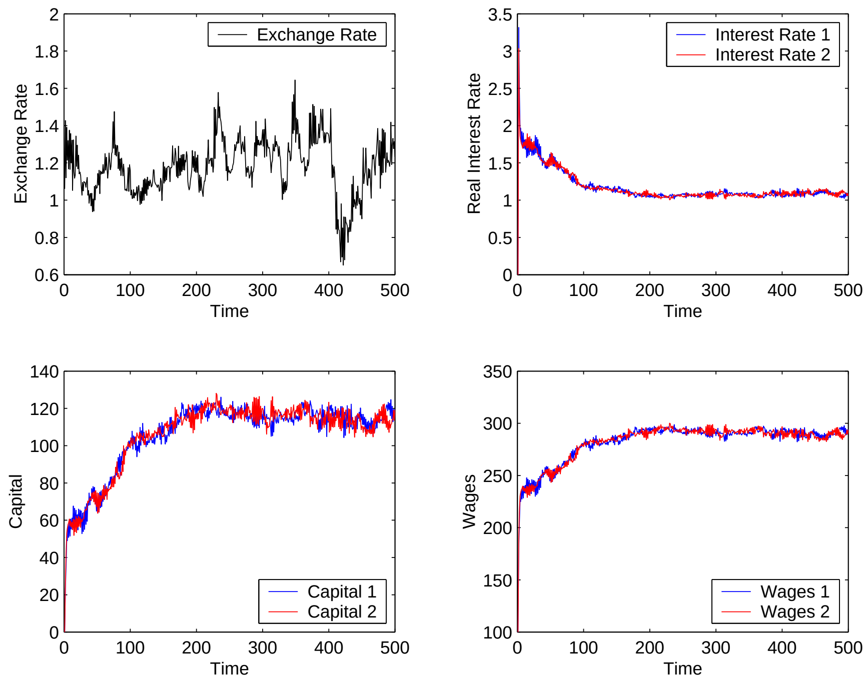


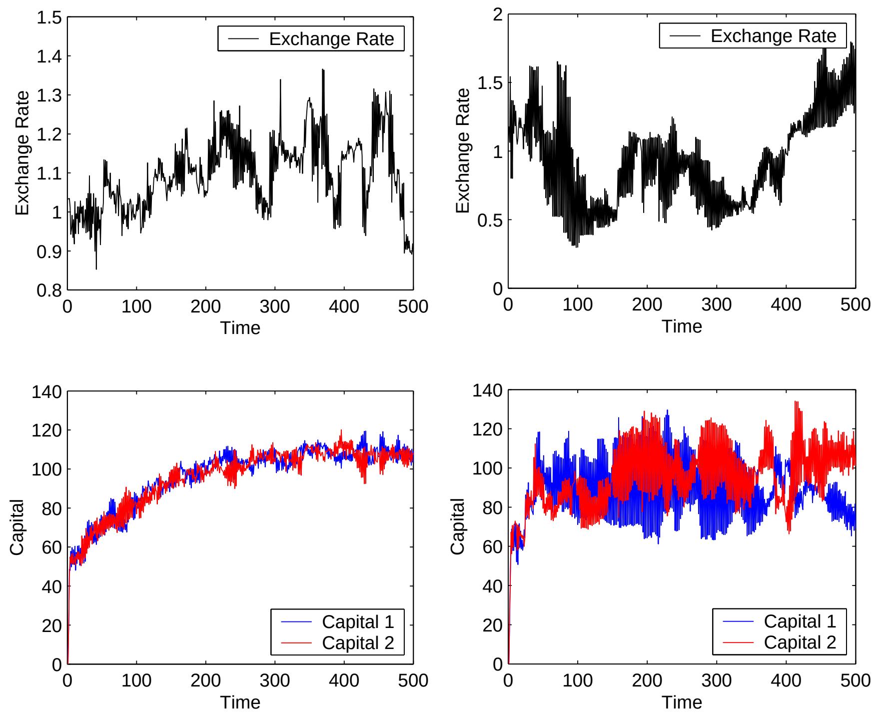
Figure 2: Baseline results, with parameterization $N = 30$, $T = 500$, $M_1 = M_2 = w_1 = w_2 = 100$, $a = b = c = d = 0.25$, and $\pi = 0.3$. Real prices (on rental and wages) converge to a quasi-steady state, with less volatility than the capital stock, which also converges. The exchange rate, however, continues to exhibit volatility persistence, with no equilibrium level.

does. More specifically, lowering the overall experimentation rate increases short run volatility substantially. At the limit, as $\pi_x \to 0$, the exchange rate oscillates between two values, switching every period. Note, as well, that the exchange rate retains the volatility persistence exhibited in the baseline.

The fact that lesser overall experimentation increases short-run volatility and inhibits convergence to the quasi-steady state, while greater experimentation alone does not, makes intuitive sense: since there is learning involved in the optimality of strategies vis-à-vis each other, differences in the initial calibration of probabilities should eventually wash out over time. In contrast, a decline in the overall experimentation rate exerts a continuous effect that is not diminished by learning over time.

In the next set of experiments we allow for variations in the calibration of initial wages. First, we first consider lowering the level of initial wages, by an order of magnitude, for both economies ($w_1 = w_2 = 10$). Next, we allow for

equally among the other decision points. These results are available on request.



(a) Greater saving experimentation

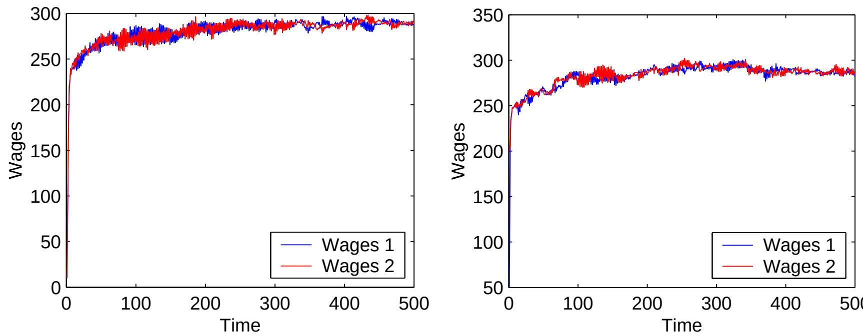
(b) Lesser overall experimentation

Figure 3: Changes to the experimentation rule. The left panel displays results for an increase in the propensity to experiment with the saving-consumption decision ($a = 0.5, b = c = d = 0.167$ with $\pi = 0.3$ same as the baseline). The right panel displays results for a decrease in the overall experimentation rate ($\pi = 0.03$, with $a = b = c = d = 0.25$ same as the baseline). Other parameters follow the baseline ($N = 30$, $T = 500$, $M_1 = M_2 = w_1 = w_2 = 100$).

the initial level of wages to diverge between the two economies, with wages four times higher in one economy relative to the other ($w_1 = 50, w_2 = 200$). As before, all other parameters follow the baseline.

Interestingly, the initial calibration of wages makes little difference to wage outcomes: in either case, wages quickly rise to oscillate about the same quasi-steady state level (Figure 4(a)). Likewise, the other endogenous variables exhibit the same patterns as before the real interest rate (capital) falls (rises) quickly to a quasi-steady state, and the exchange rate remains volatile (Figure 4(b)). We conclude that the divergence in the relative behavior of the exchange rate and the real economy is little affected by changes in initial real-side prices, at least within our artificial economic setting.

Out third set of experiments allows for variations in the calibration of initial



(a) Low initial wages

(b) Different initial wages

Figure 4: Changes to wage calibration. The left panel displays results for low levels of initial wages in both economies ($w_1 = w_2 = 10$). The right panel displays results for differing initial wages in the two economies ($w_1 = 50, w_2 = 200$). Other parameters follow the baseline ($N = 30$, $T = 500$, $M_1 = M_2 = 100$, $a = b = c = d = 0.25$, $\pi = 0.3$).

money supplies. As was the case for wages, we consider lowering the level of initial money supply, by this time to two orders of magnitude, for both economies ($M_1 = M_2 = 1$). Then, as before, we allow for the initial levels to diverge, with money supply a hundred times greater in one economy relative to the other ($M_1 = 100, M_2 = 1$). We choose these larger magnitudes in order to accentuate the possibility that divergences matter; the rest of the parameters otherwise follow the baseline.

The results echo the case for wages: there is little qualitative effect from changes in initial money supply.[6] While the fact that the initial calibration of nominal values (the money supply) makes little difference to the outcome is not entirely surprising—especially in light of the Kareken & Wallace (1981) indeterminacy result—the imperviousness of real variables (real wages) may, at first, seem more surprising. However, more careful inspection of the no-arbitrage condition (5) hints at why this could be the case: there is, ultimately, an equivalency in the real and nominal quantities over time. While (5) describes the perfect-foresight benchmark, it is conceivable that equilibrium outcomes even in our agent-based model would tend toward such an equilibrium.

Our final set of experiments selectively suppresses one of the three genetic algorithms, and examines how doing so alters the baseline.[7] In this case, although the exclusion of the imitation algorithm does not alter the qualitative results in any substantive way, the absence of either experimentation or elective gives rise to extremely volatile, oscillating variables after around a hundred periods. This volatility is similar to, but even more severe than, the reduction of

[6]The relevant charts are provided in the online appendix.
[7]The relevant charts are likewise provided in the online appendix.

the experimentation rate. More importantly, there is a failure of convergence between the interest rate, wages, or capital stock. These experiments point to the importance of at least the latter two genetic algorithms for our results.

5 Conclusion

The objective of this paper has been to consider, in the context of a two-country, overlapping-generations model, whether stable net real capital flows can coexist with a volatile nominal exchange rate. To that end, we embed capital into the monetary economies of Arifovic (2001), and while we replicate the main result of that paper—volatility persistence in the exchange rate—we also find that this occurs *in spite of* convergence in the real economy to a quasi-steady state. Moreover, as the mutation rate tends to zero, the variance in the exchange rate becomes concentrated in the short term, due to the collapse of the population of strategies.

The main practical implication that arises from this work is straightforward. There has been a recent revival among academic and policy circles in imposing stronger regulation and control over cross-border capital flows (Moore 2014; Ostry, Ghosh, Chamon & Qureshi 2011). While such controls are often regarded as a means to limit the worst excesses of volatility in the macroeconomy and financial markets, our work here suggests that while this may well be true for the real economy, there are reasons to believe that financial markets will continue exhibit volatile characteristics, independent of relative real macroeconomic stability.

Given our interest in the effects of the real side of the economy on the exchange rate, a major concern arising from this model is the somewhat limited interaction between the real and the monetary aspects of the economy. The first step for future research is to bridge this gap. Possible extensions include endogenizing the labor decision, adding a transactions demand for money, allowing for longer-lived agents, and adding more microstructure to production (perhaps monopolistically competitive production with firm-specific investment in individual goods varieties). These extensions could be construed as further checks on the robustness of the endogenous persistence of exchange rate volatility.

References

Andersen, Torben G. & Tim P. Bollerslev (1997). "Intraday Periodicity and Volatility Persistence in Financial Markets". *Journal of Empirical Finance* 4(2–3) (June): 115–158

Andersen, Torben G., Tim P. Bollerslev, Francis X. Diebold & W. Paul Labys (2001). "The Distribution of Realized Exchange Rate Volatility". *Journal of the American Statistical Association* 96(453) (March): 42–55

Arifovic, Jasmina (1996). "The Behavior of the Exchange Rate in the Genetic Algorithm and Experimental Economies". *Journal of Political Economy* 104(3) (June): 510–541

Arifovic, Jasmina (2001). "Performance of Rational and Boundedly Rational Agents in a

Model with Persistent Exchange Rate Volatility". *Macroeconomic Dynamics* 5(2) (April): 204–224

Baxter, Marianne & Alan C. Stockman (1989). "Business Cycles and the Exchange-Rate Regime: Some International Evidence". *Journal of Monetary Economics* 23(3) (May): 377–400

Bollerslev, Tim P., Ray Y. Chou & Kenneth F. Kroner (1992). "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence". *Journal of Econometrics* 52(1–2) (April–May): 5–59

Chari, Varadarajan V., Patrick J. Kehoe & Ellen R. McGrattan (2002). "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?" *Review of Economic Studies* 69(3) (July): 533–563

Cont, Rama (2007). "Volatility Clustering in Financial Markets: Empirical Facts and Agent-Based Models". In Alan P. Kirman & Gilles Teyssière (editors), *Long Memory in Economics*, pp. 289–309. Heidelberg, Germany: Springer

Engel, Charles A. (1994). "Can the Markov Switching Model Forecast Exchange Rates?" *Journal of International Economics* 36(1–2) (February): 151–165

Engel, Charles A. & James D. Hamilton (1990). "Long Swings in the Dollar: Are They in the Data and Do Markets Know It?" *American Economic Review* 80(3) (September): 689–713

Engle, Robert F. (2001). "GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics". *Journal of Economic Perspectives* 15(4) (Fall): 157–168

Kareken, John H. & Neil W. Wallace (1981). "On the Indeterminacy of Equilibrium Exchange Rates". *Quarterly Journal of Economics* 96(2) (May): 207–222

Lee, Hsiu-Yun & Show-Lin Chen (2006). "Why Use Markov-Switching Models in Exchange Rate Prediction?" *Economic Modelling* 23(4) (July): 662–668

Mandelbrot, Benoît B. (1963). "The Variation of Certain Speculative Prices". *Journal of Business* 36(4) (October): 394–419

Monacelli, Tommaso (2004). "Into the Mussa Puzzle: Monetary Policy Regimes and the Real Exchange Rate in a Small Open Economy". *Journal of International Economics* 62(1) (January): 191–217

Moore, Winston R. (2014). "Managing the Process of Removing Capital Controls: What Does the Literature Suggest?" *Journal of Economic Surveys* 28(2) (April): 209–237

Mussa, Michael L. (1986). "Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates: Evidence and Implications". *Carnegie-Rochester Conference Series on Public Policy* 25(1) (January): 117–214

Ostry, Jonathan D., Atish R. Ghosh, Marcos Chamon & Mahvash S. Qureshi (2011). "Capital Controls: When and Why?" *IMF Economic Review* 59(3) (August): 562–580

Rogoff, Kenneth S. (1999). "Perspectives on Exchange Rate Volatility". In Martin Feldstein (editor), *International Capital Flows*, pp. 441–453. Chicago, IL: University of Chicago Press

Sargent, Thomas J. (1993). *Bounded Rationality in Macroeconomics*. Oxford, England: Oxford University Press